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# Finite element analysis of pulsed field magnetization process in a cylindrical bulk superconductor

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ABSTRACT

The magnetization properties in a cylindrical superconductor with an infinite length exposed to a pulsed external magnetic field are numerically evaluated by means of a coupled finite element method. In order to estimate the distributions of magnetic field and temperature inside the superconductor, magnetic diffusion and heat balance equations are alternately and iteratively solved at each time step. It is assumed that the superconductor has a transport property represented by the power-law model, where the critical current density depends on the local temperature. The adiabatic condition for the thermal analysis is also used, so that the temperature rise comes from the local energy dissipation due to the magnetic flux motion. The influences of the strength of external applied field and the initial temperature in the superconductor on the final trapped magnetic field are investigated toward an optimal design of bulk superconductor magnet.



## 1. Introduction

There are two major processes for the magnetization of a bulk superconductor material. One is a field cooling method, which requires a relatively large system to supply a stable magnetic field. The other is a pulsed field magnetization (PFM) method [1], which can be realized inexpensively with a copper coil and a capacitor bank. In the latter technique, however, the magnitude of applied field has to be chosen carefully from both viewpoints of a temperature rise and a final trapped magnetic field in the superconductor [1–3].

In the PFM methods, the multiple applications of external fields to the bulk superconductor are usually carried out to improve the final trapped magnetic field. The typical methods with different sequences of external applied field and cooling temperature are illustrated in Fig. 1. Fig. 1a represents a successive magnetic pulse application (SPA) method [4] that requires several times of applying the external magnetic fields with almost same strength under a constant cooling temperature. In a modified multi-pulse technique with step-wise cooling (MMPSC) method [5,6] shown in Fig. 1b, on the other hand, two kinds of cooling temperatures are used effectively. First, the pulsed magnetic fields with relatively small magnitude are applied twice to the bulk superconductor cooled around a relatively high temperature. After that, the superconductor is cooled down to a lower temperature and the applications of a few pulsed fields with larger strength are performed intermittently. By using such a MMPSC process, a high trapped magnetic field of 4.47 T has been achieved at the center of a Gd-based bulk superconductor in the form of a disk [5]. However, the mechanism of magnetization process for the MMPSC method has not been clarified yet.

In the present study, the magnetization properties in a cylindrical superconductor exposed to multiple pulsed magnetic fields are numerically evaluated by means of a finite element method. The electromagnetic and thermal analyses are coupled with each other to take into account the effect of heating due to the penetration of magnetic flux into the superconductor on the final trapped field. The obtained numerical results are compared with the experimental data published already [5], and the effectiveness of the MMPSC method is confirmed qualitatively.

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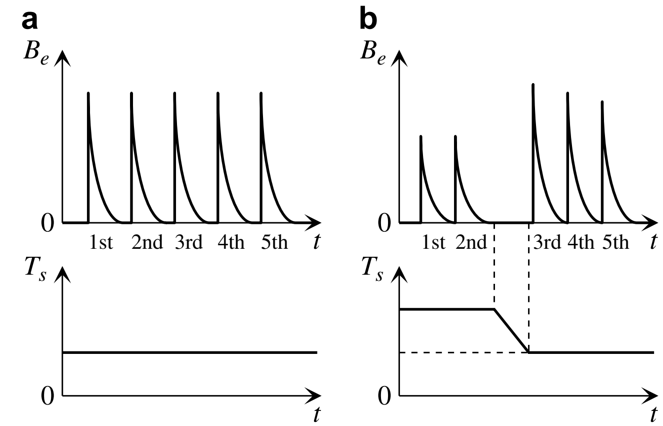


**Fig. 1.** Schematic diagrams of profiles of external applied magnetic field and cooling temperature for pulsed field magnetization techniques with (a) SPA method and (b) MMPSC method.

## 2. Numerical analysis of PFM process

In order to evaluate the magnetic flux motion and the temperature rise in a bulk superconductor exposed to an external magnetic field, their numerical simulation is carried out as follows. In spite of the use of a Gd-based bulk superconductor disk with diameter of 45 mm and thickness of 15 mm in the experiment [5], let us consider an infinitely long cylinder with the identical diameter for the sake of simplification. Although such assumption is unsuitable for quantitative evaluation, the advantages in the MMPSC method are reconfirmed numerically in this paper. If the local magnetic field is given by the sum of a uniform external field $B_e$ applied in the axial direction and a field $B$ generated by a current induced in the azimuthal direction, the governing equation for the electromagnetic field can be obtained with Maxwell's equations [7],

$$\frac{1}{\mu_0}\frac{1}{r}\frac{\partial}{\partial r}\left(\rho r\frac{\partial B}{\partial r}\right)=\frac{\partial B}{\partial t}+\frac{\partial B_e}{\partial t}, \qquad (1)$$

where $\rho$ is the equivalent resistivity of the superconductor, which is represented here with the power-law model for the voltage–current characteristics as

$$\rho(J,T)=\frac{E_c}{J_c(T)}\left[\frac{|J|}{J_c(T)}\right]^{n-1}, \qquad (2)$$

where $E_c$ is the electric-field criterion of the critical current density $J_c$ and $n$ the $n$-value. As can be seen in Eq. (2), the resistivity of superconductor is generally a function of the local current density $J$ and the temperature $T$. The linear relationship in the critical current density versus temperature is assumed here as

$$J_c(T)=J_{c0}\frac{T_c-T}{T_c-T_0}, \qquad (3)$$

where $T_c$ is the critical temperature and $J_{c0}$ the critical current density at the reference temperature $T_0$. In addition, the field dependence of the critical current density is not focused on because the effect of the decrease in the critical current density due to only the temperature rise on the final trapped field is revealed. The following fixed parameters are used: $T_c = 93$ (K), $E_c = 1$ (μV/cm), $J_{c0} = 20$ (kA/cm$^2$) at $T_0 = 40$ (K) and $n = 15$.

The governing equation for thermal analysis in the model under consideration is represented by the heat balance equation,

$$C\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(\kappa r\frac{\partial T}{\partial r}\right)+\rho J^2, \qquad (4)$$

where $C$ is the heat capacity and $\kappa$ the thermal conductivity. The term of cooling on the right-hand side in Eq. (4) is disregarded in advance although the surface of bulk superconductor has been cooled with a refrigerator in the experiment [5], but its treatment in this study is explained in the first part of the following section. The thermal properties as a function of temperature, $C(T)$ and $\kappa(T)$, are taken into account on the basis of the experimental results for the Gd-based bulk superconductor [8].

The above-mentioned governing equations for the electromagnetic and thermal analyses have an analogous structure, so that $B$, $\rho/\mu_0$, 1 and $\partial B_e/\partial t$ in Eq. (1) correspond to $T$, κ, $C$ and $(-\rho J^2)$ in Eq. (4), respectively. This means that the programming code for one analysis is applicable to the other without major modification. The governing equations are discretized by means of the Galerkin method and the backward difference method for space and time, respectively. The obtained two kinds of simultaneous equations are alternately solved at each time step. Since iterative calculations at fixed time are also needed for the physical parameters depending on the unknown variables, the successive under-relaxation method is applied with the relaxation factor of 0.1 [9]. Further-

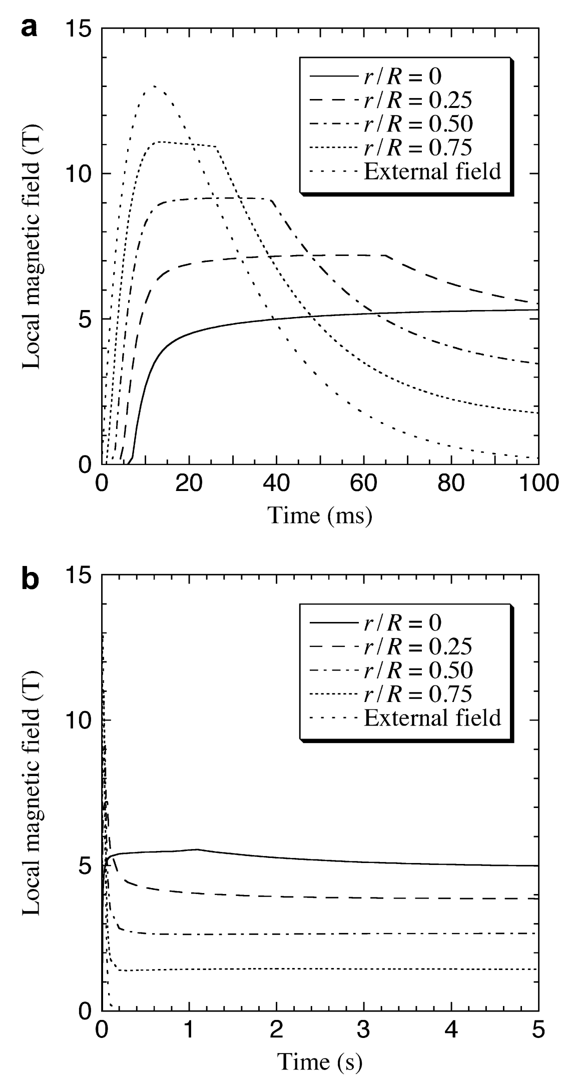


**Fig. 2.** Time dependence of local magnetic fields at fixed positions inside superconductor cylinder. (a) Focuses mainly on a period for the pulsed field application, whereas the convergence of field distribution is confirmed for a longer time scale in (b). The peak of external field and the initial temperature are 13.0 T and 30 K, respectively.

more, the waveform of the external magnetic field $B_e$ generated with a pulse coil installed in a capacitor discharge circuit is approximated by [8]

$$B_e(t) = B_m\left[\left(\frac{\tau_1}{\tau_2}\right)^{\frac{\tau_1}{\tau_2-\tau_1}} - \left(\frac{\tau_1}{\tau_2}\right)^{\frac{\tau_2}{\tau_2-\tau_1}}\right]^{-1}\left(e^{-\frac{t}{\tau_2}} - e^{-\frac{t}{\tau_1}}\right), \tag{5}$$

where $B_m$ is the peak of the external applied field. In Eq. (5), $\tau_1$ and $\tau_2$ represent the decay time constants, and they are estimated as $\tau_1 = 8$ (ms) and $\tau_2 = 19$ (ms) for the first rising time of about 12 ms [8]. The boundary conditions for both analyses are set to

$$B(R,t) = B_e(t), \tag{6}$$

$$T(R,t) = T_s, \tag{7}$$

where $R$ and $T_s$ are the radius and cooling temperature of superconductor cylinder, respectively.

## 3. Numerical results and discussion

Fig. 2 shows a typical example of the time dependence of local magnetic fields at several fixed positions inside the superconductor cylinder. The peak of external magnetic field and the initial temperature are 13.0 T and 30 K, respectively. It can be seen in Fig. 2a that the local magnetic fields rise up in sequence with increasing the external applied field and then a part of the penetrating magnetic flux gets out of the superconductor for the decrease in the external field. After the pulsed field application, the local magnetic fields asymptotically approach to their own constants, and it typically takes about 5 s for their convergence, as shown in Fig. 2b. In the refrigerated superconductor under consideration, the thermal diffusivity $\kappa/C$ is much smaller than the magnetic diffusivity $\rho/\mu_0$, and therefore their diffusion times are quite different from each other. Although a few tens of minutes seem to be needed for uniform cooldown of a bulk superconductor with a cryocooler after applying a pulsed field in the experiment [5], the numerical calculation under the adiabatic condition is carried out up to 5 s and the temperature inside the superconductor is initialized to uniform distribution with the field profile unchanged in order to save a computation time.

The dependence of magnetic field trapped at the center of superconductor cylinder on the peak of external applied field is shown in Fig. 3a, where the pulsed field is applied only once. It can be found that there is an optimum magnitude of external field

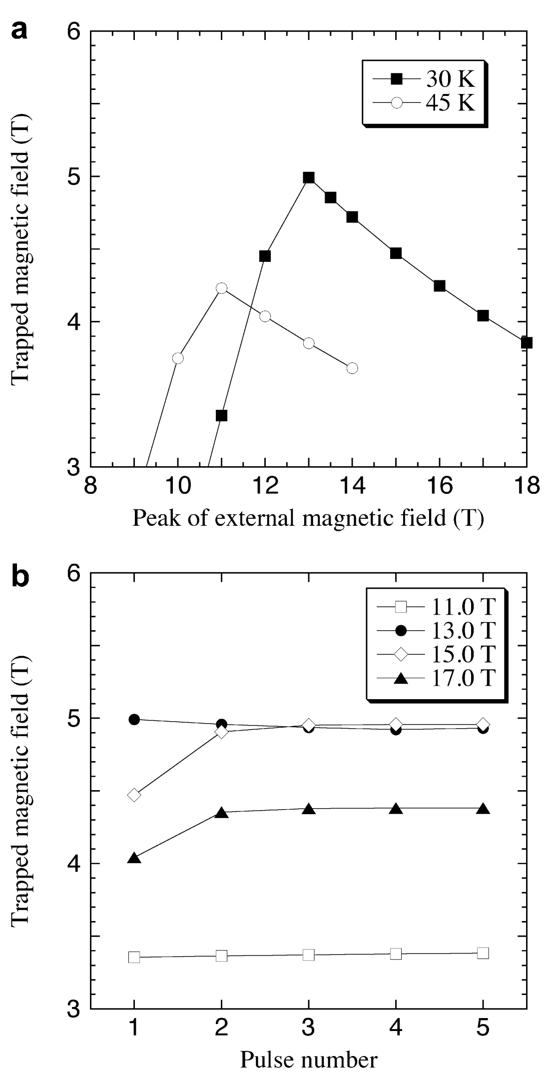


**Fig. 3.** Numerical results of trapped magnetic fields in superconductor cylinder for (a) single pulse and (b) SPA method. The cooling temperature for the SPA method is fixed at 30 K.

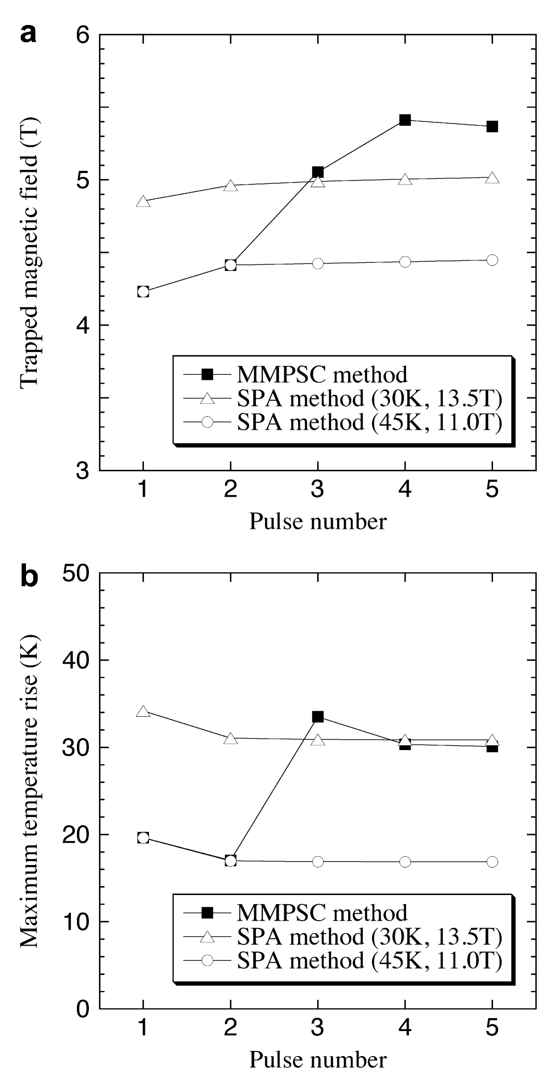


**Fig. 4.** Pulse number dependence of (a) trapped magnetic field and (b) maximum temperature rise for SPA and MMPSC methods. The similar properties obtained in the experiment have already been presented [5].

that the final trapped field becomes a maximum. Such a property shifts to a larger range of magnetic field with decreasing the initial cooling temperature. Fig. 3b shows the numerical results of the trapped magnetic field in the superconductor at each step of pulsed field application for the SPA method with the cooling temperature of 30 K. It can be seen that the trapped fields except for first pulse application scarcely vary, and that a relatively broad range of the external field for the maximum of final trapped field is observed as compared with the results for the single pulse given in Fig. 3a.

Fig. 4 shows the comparison between the numerical results of trapped magnetic field and maximum temperature rise at each step for multiple PFMs with the SPA and MMPSC methods. In the MMPSC method, the cooling temperature and the external applied field strength in the first two steps are fixed at 45 K and 11.0 T, respectively. After that, the cooling temperature goes down to 30 K. The magnitudes of external fields for third, fourth and fifth pulses are also set to 14.5, 13.5 and 13.5 T, respectively. It is found in Fig. 4a that the largest trapped magnetic field is obtained at fourth application of pulsed field in the MMPSC method. It is also seen in Fig. 4b that the maximum temperature rises in the MMPSC method are almost same as those for the SPA method under the identical conditions expect for third pulse application. It has to be pointed out that the properties of trapped magnetic field and maximum temperature rise given in Fig. 4 agree qualitatively with the experimental results [5]. This means that the present numerical simulation is quite valid to understand the advantages in the MMPSC method compared with the SPA method although the assumption of the superconductor cylinder with infinite length differs from an actual shape of specimen in the experiment [5].

## 4. Conclusions

The numerical simulation of the pulsed field magnetization in the cylindrical superconductor was carried out with the coupled finite element method for electromagnetic and thermal analyses. It was assumed that the superconductor cylinder had an infinite length and the magnetic field trapped at the center finally for the pulsed field application was evaluated as well as the radial distributions of magnetic field and temperature inside it. By taking into account the power-law model for the voltage–current characteristics with the temperature dependence of critical current density, the similar properties for final trapped magnetic field and maximum temperature rise as the experiment were obtained numerically. The further investigation for more quantitative evaluation of the pulsed field magnetization process will be needed toward an optimal design of bulk superconductor magnet.

## References

[1] See for example Y. Itoh, U. Mizutani, Jpn. J. Appl. Phys. 35 (1996) 2114.
[2] M. Tsuchimoto, K. Morikawa, IEEE Trans. Appl. Supercond. 9 (1999) 66.
[3] S. Bræck, D.V. Shantsev, T.H. Johansen, Y.M. Galperin, J. Appl. Phys. 92 (2002) 6235.
[4] H. Fujishiro, K. Yokoyama, M. Kaneyama, T. Oka, K. Noto, Physica C 412–414 (2004) 646.
[5] H. Fujishiro, M. Kaneyama, T. Tateiwa, T. Oka, Jpn. J. Appl. Phys. 44 (2005) L1221.
[6] H. Fujishiro, T. Tateiwa, T. Hiyama, Jpn. J. Appl. Phys. 46 (2007) 4108.
[7] K. Kajikawa, T. Hayashi, R. Yoshida, M. Iwakuma, K. Funaki, IEEE Trans. Appl. Supercond. 13 (2003) 3630.
[8] H. Fujishiro, M. Kaneyama, K. Yokoyama, T. Oka, K. Noto, Supercond. Sci. Technol. 18 (2005) 158.
[9] T. Nakata, N. Takahashi, Finite Element Method in Electrical Engineering, second ed., Morikita Publishing, Tokyo, 1986. p. 195 (in Japanese).